Avalanches in draining foams

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LETTER TO THE EDITOR

Avalanches in draining foams

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Abstract. We present a first statistical analysis of the noise made by the bursting cells of a draining foam. We show that, for old foams i.e. for foams with cells limited by very thin films, the cell ruptures are correlated and form avalanches. We propose some possible origins for this correlation.

1. Introduction

The ageing of aqueous foams can easily be observed, for instance in a glass of beer or in a sink while doing the washing-up. Several stages can be distinguished in the life of the foam.

(a) Initially, the foam is a dispersion of *spherical* gas bubbles in a liquid (*wet* foam).
(b) Gravity then causes the bubbles to rise and the liquid flows down: the bubbles cluster to form a polyhedral structure of cells (polyaphron). The liquid is pumped into the Plateau borders (the regions where the cell walls meet) because of the negative Laplace pressure of their concave surfaces; the films become thinner and the foam becomes *dry*. During this stage, the structure of the foam changes, eliminating the mechanical constraints by sliding motions of the cells walls (T_1 process).
(c) Eventually, the air enclosed in the cells diffuses from the high pressure cells towards the low pressure cells (T_2 process): the number of cells thus decreases.

This last stage is very slow since it is limited by the gas diffusion across the soap films (water-swollen surfactant bilayers) and the disappearance of cells is usually due to a completely different process: the walls of the cells of an aged foam are very thin elastic sheets that can be broken by any faint perturbation. Moreover, it is usually observed that bubbles then break in a collective mode. This observation led us to the idea of describing the last moments of the existence of a foam by a succession of avalanches triggered by single rupture events. In this letter, we study the statistics of the bubble bursts by recording the sound emitted by the cell ruptures. Our primary goal is to investigate whether an aqueous foam could provide a model to study dynamic collective phenomena and avalanches [1, 2], as first proposed with magnetic garnet films [3–5] that behave as 2D foams. The study of the detailed acoustic signature of the bursting events also constitutes an interesting field of investigation but is out of the scope of the present work.

2. Experimental set-up

The foaming agent was sodium dodecyl sulphate (SDS), an anionic surfactant purchased from Prolabo (Paris, France) and used as received. It was dissolved in triply distilled water. In

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order to investigate the possible relevance of the Gibb's elasticity, the concentration was varied between 0.13 and 1.00 cmc. The critical micellar concentration (cmc) is the concentration above which SDS molecules self-assemble into spherical aggregates to minimize the number of contacts between water and aliphatic chains ($\mathrm{cmc_{SDS}} = 2.4$ g l^{-1}) [6]. We have also used glycerol to increase the bulk viscosity of the soap solutions.

The foams are made in a fish tank ($30 \times 20 \times 20$ cm^3) by blowing air into the soap solution through a piece of sintered glass. The average size of the generated bubbles is about 1 mm and the initial height of the foam floating on top of the soap solution is typically 10 cm: the number of cells at the beginning of the experiment is thus of order 10^7. The sounds emitted by the ruptures of the bubbles were detected by means of an ordinary electret microphone (the type used in telephone handsets) hanging above the top surface of the foam. The signal supplied by the microphone is amplified and filtered by a 3 kHz high-pass filter; it is then sampled at a rate of 10 kHz with an 8-bit accuracy and stored on the hard disk of a microcomputer. Data are preselected during sampling and storing in order to handle files of reasonable size: an additional digital high-pass filtering is performed (we used a fourth-order Butterworth algorithm with a 1250 Hz frequency [7]) and only values $S(k)$ above a fixed threshold are stored. This processing has the essential advantage of not introducing artificial correlation between the bursting events.

After the experiment, a peak extraction is performed. For this an envelope $E(k)$ of the raw signal $S(k)$ is calculated

$$E(k) = \max(|S(mk)|, \ldots, |S(m(k+1)-1)|). \tag{1}$$

We took $m = 20$. Peaks associated with bubble ruptures are then determined by the local maxima of $E(k)$.

3. Results

Figures 1 and 2 represent on a semi-logarithmic scale the frequency of bubble ruptures (actually the average number of ruptures for five seconds of counting) as a function of time after formation. The foams are made out of two solutions with different soap concentrations, namely a 0.13 cmc solution giving very unstable foams and a 0.25 cmc solution forming more stable foams. Three regimes can be distinguished.

(a) After a short transition period (for the 0.13 cmc sample), the rate of bubble rupture decays exponentially with time t as $\exp(-\lambda t)$. We may reasonably assume that in this first regime, no correlation between bubble ruptures should take place because of the wet character of the foam. Within this assumption, λ^{-1} is the mean lifetime of a single bubble and is about 1500 s for the 0.13 cmc foam and 3900 s for the 0.25 cmc foam. This difference might be attributed to a demonstration of a Marangoni effect [8] that slows down the liquid flow between the surfactant walls.

(b) A quiet regime then follows where the bubble ruptures are quite rare.

(c) Eventually, there is an important increase in rupture activity before the total disappearance of the foam. This last regime is quite important for the 0.25 cmc foam with respect to the 0.13 cmc foam (in the latter case, this late regime is almost nonexistent). This final increase of the rupture rate is a common observation: a foam (for instance a foam produced in a sink to do the washing-up) usually drains slowly for a while and then suddenly breaks to eventually disappear.

These observations led us to investigate whether this last stage could represent a cooperative phenomenon i.e. if a single bubble rupture could trigger off other ruptures and

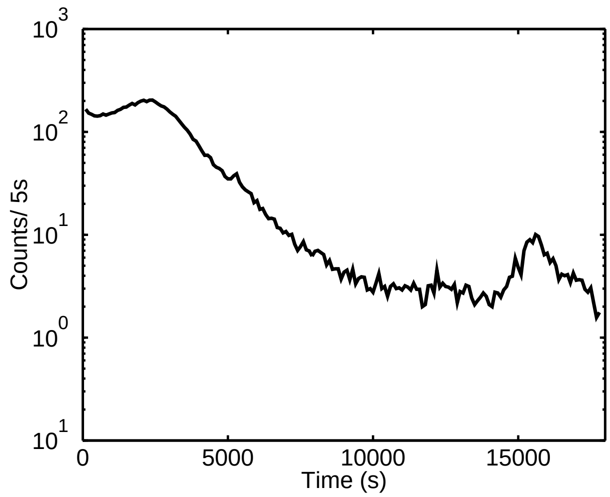

Figure 1. Evolution with time of the frequency of bubble ruptures for a foam made from a 0.13 cmc SDS solution (data have been averaged over 100 seconds).

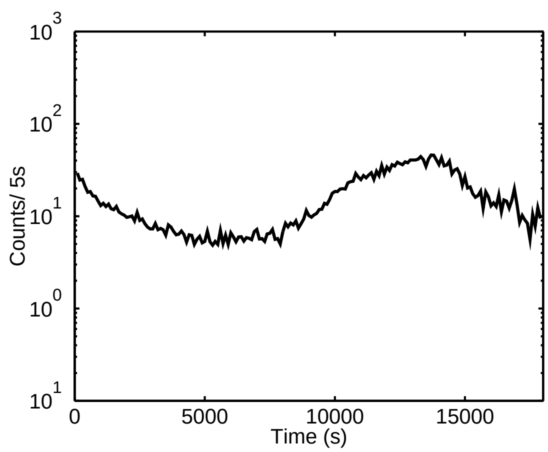

Figure 2. Evolution with time of the frequency of bubble ruptures for a foam made from a 0.25 cmc SDS solution (data have been averaged over 100 seconds).

possibly an avalanche of ruptures. Each processed experiment gives an array (t_i, a_i) for the dates and the amplitudes of the peaks. As stated above, our measurement line is not sophisticated enough to study the acoustic signature of a single burst with great confidence, and since we are mainly concerned with the possible correlation between events, we concentrate on the time series t_i.

In the steady state, in the case of uncorrelated events, one would expect the number of peaks in a time interval to have a Poisson distribution: the probability of finding n peaks within an interval with an expected number of peaks $\mu = \langle n \rangle$ should then be given by

$$p_n = e^{-\mu} \frac{\mu^n}{n!}. \tag{2}$$

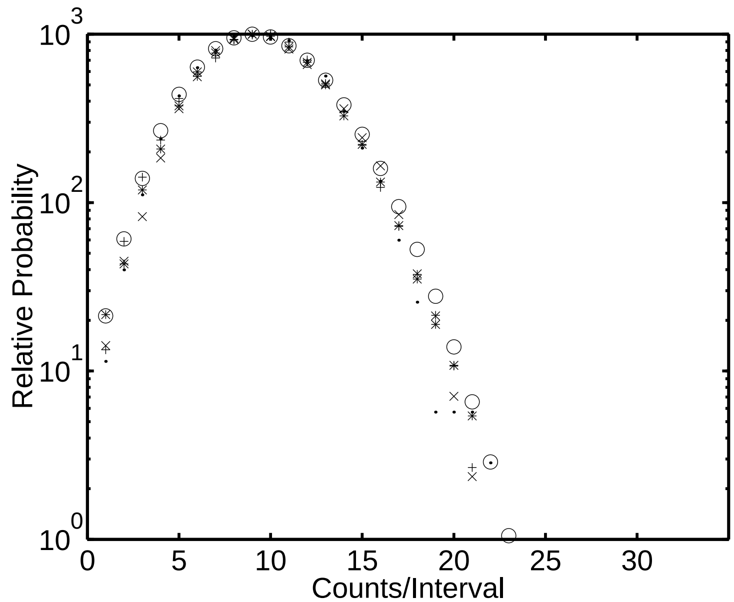

Figure 3. Poisson comparison plots for the 0.13 cmc foam of the 0–250 (×), 250–500 (⋇), 500–750 (·) and 750–1000 (+) seconds of the life of the foam. ○ represents the Poisson distribution given by equation (2).

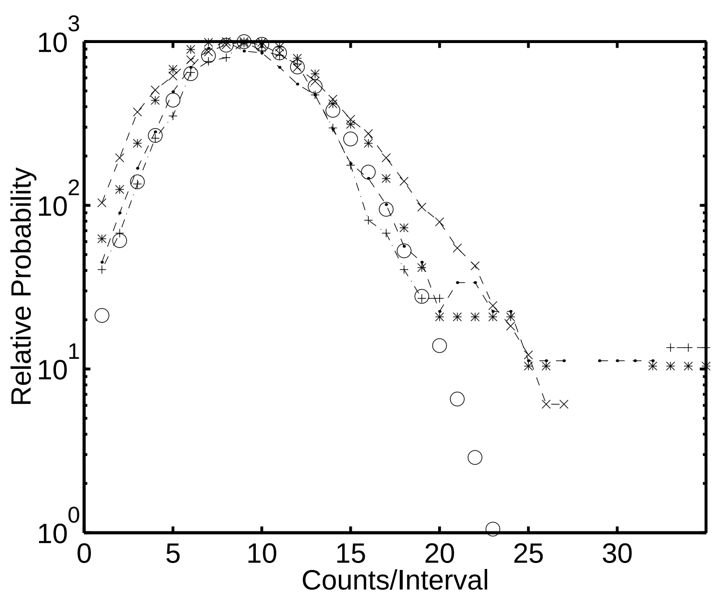

Figure 4. Poisson comparison plots for a young 0.25 cmc foam of the 0–1000 (×), 1000–2000 (⋇), 2000–3000 (·) and 3000–4000 (+) seconds of the life of the foam. ○ represents the Poisson distribution given by equation (2).

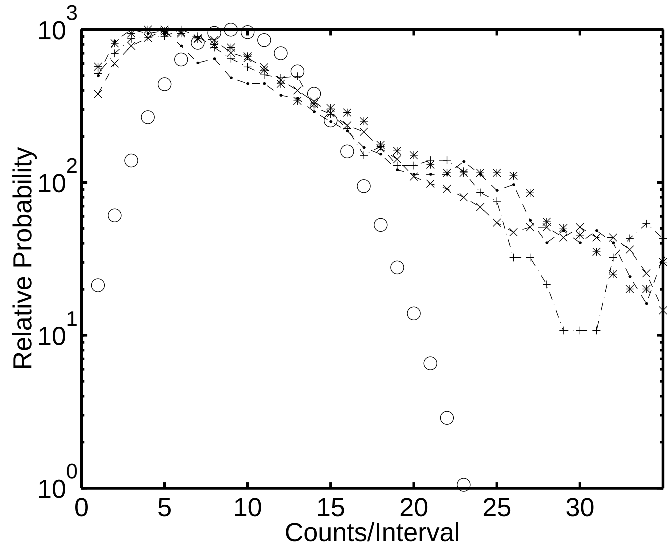

Figure 5. Poisson comparison plots for an old 0.25 cmc foam of the 13000–14000 (×), 14000–15000 (⋇), 15000–16000 (·) and 16000–17000 (+) seconds of the life of the foam. ○ represents the Poisson distribution given by equation (2).

A deviation from this Poisson distribution should then disclose correlations. We thus divide the duration of a whole experiment into large time *windows* of 1000 seconds. These windows are then partitioned into smaller, equal-sized *intervals* whose length is set to get an maximum expected number of peaks equal to 10. We paid great attention to checking that we were not introducing artificial correlation during this process; in particular we took great care to check that the different rates of bubbles rupture at the beginning and end of each window did not induce artificial deviations from the Poisson distribution. In figures 3–5 we compare the distribution of the bubble ruptures with a Poisson-type behaviour (given by equation (2)) for the 0.13 cmc and 0.25 cmc foams. More precisely, we plot the relative probability of observing intervals including k peaks (events) against k. All curves have been normalized to ensure the same maximum value for the purpose of comparison, and the scattering of the data gives an estimate of accuracy of the measurements. We can remark that the least stable foam (0.13 cmc) closely obeys the Poisson law after its formation; this foam was so unstable that we could not investigate the distribution of the ruptures after the first regime, as we discussed earlier. The behaviour of the other foam (0.25 cmc) is close to the Poisson at short times in the first regime (figure 4), but exhibits strong deviations during the last regime of rupture resumption (figure 5), thus proving the existence of correlations between ruptures in the *dry* foam. We have always observed this final increase of correlations in old metastable foams; the reproducibility depends on the control of the air humidity in the environment of the fish tank. For instance, we were able to trigger *giant* avalanches by suddenly removing a lid that kept the foam in the fish tank water-saturated.

4. Simulations

In order to investigate the effect of cooperative phenomena on the bubble rupture distribution, we have performed a rather naive and very simple 2D computer simulation. The purpose of this simulation is not to develop another theory for avalanches, but to illustrate how correlations causing avalanches may lead to deviations from the Poisson distribution. It is assumed that the films of the foam can be stressed up to a certain threshold (the yield stress S_y) before they break. When an individual film breaks, the stress of the film is transferred to its neighbours: if this additional stress causes one of the neighbours to break, the sum of the stresses is transferred to the next neighbours, and so on. More precisely, in the beginning of the simulation, a honeycomb network (with periodic boundary conditions) of films is created. Each film F_i is assigned to a random actual stress S_i, with $(0 < S_i < S_y)$. The simulation is run by randomly choosing films that break and transfer their stress to their neighbours. Only a fraction of the stress S_i of a broken film is transferred to its neighbours. The reason is that, if the entire stress were transferred, $\sum_{F_i} S_i = \mathrm{const.}$ (i.e. the stress in the system remains constant) while $\sum_{F_i} S_y$, i.e. the ability of the entire system to endure stress decreases with the number of cells. This results in catastrophic events after a small number of steps. By introducing a dissipation coefficient (i.e. by transferring only a fraction of the stress; this could reflect the rearrangement of cells around a burst site) one can obtain more realistic, stationary scenarios that also exhibit avalanches. We define the size s of an avalanche by the number n_{ind} of induced ruptures by the trigger event plus one ($s = n_{ind} + 1$). This simulation yields a $1/f^{\alpha}$-type avalanche size distribution with the α exponent equal to 1.75. We were not able to distinguish a clear $1/f$ behaviour from the spectral density of our experimental time series t_i: the main reason is that several avalanches can develop simultaneously in real foams. We have thus considered Poisson comparison plots (as in figures 3–5) derived from our simulations and compared them with the experimental Poisson comparison plots.

We assumed a sequence of avalanches with $1/f^{\alpha}$ size distribution. The onsets of the avalanches are uncorrelated and we set a maximum size β of avalanche (β is the maximum number of cell ruptures within a single avalanche). The events of a particular avalanche do not happen simultaneously: this is modelled by increasing the probability of an event during an avalanche by a factor γ ($\gamma = \infty$ thus corresponds to avalanches where the β ruptures occur simultaneously). α, β and γ have been varied. The simulated peak sequences yield Poisson comparison plots (see figure 6 for instance), which are indeed similar to those obtained experimentally, although of course no quantitative use of the parameters α, β and γ can reasonably be made.

5. Conclusions

Foams are good candidates for the investigation of cooperative dynamic phenomena. Avalanches of T_1 processes have already been described both experimentally [9] and theoretically [10] in the shearing of foams, and our experiments show the relevance of rupture avalanches in the ageing of foams. The coupling of the rupture events increases towards the end of the foam life when the films are thinner, and this suggests that this coupling is likely to be due to the elastic character of the cell walls: this explains why the 0.13 cmc foam is not avalanche-prone (its stability is too small to form an extended dry foam) and why we observe that the addition of glycerol increases the occurrence of avalanches since it allows us to obtain older (and thus drier) foams. Additional experiments that are not reported in this letter have been completed in 2D (using bubble rafts [11]) in order to increase the correlations [12]. They have shown that the rupture correlation increases when the soap concentration increases from 0 to 0.5 cmc, suggesting that the elastic coupling is certainly due to the Gibb's elasticity, which shows the same trend [8, 13]. Nevertheless, more experiments are needed to quantify this coupling and to link it to the recent determination of the elastic constants of a single soap film [14].

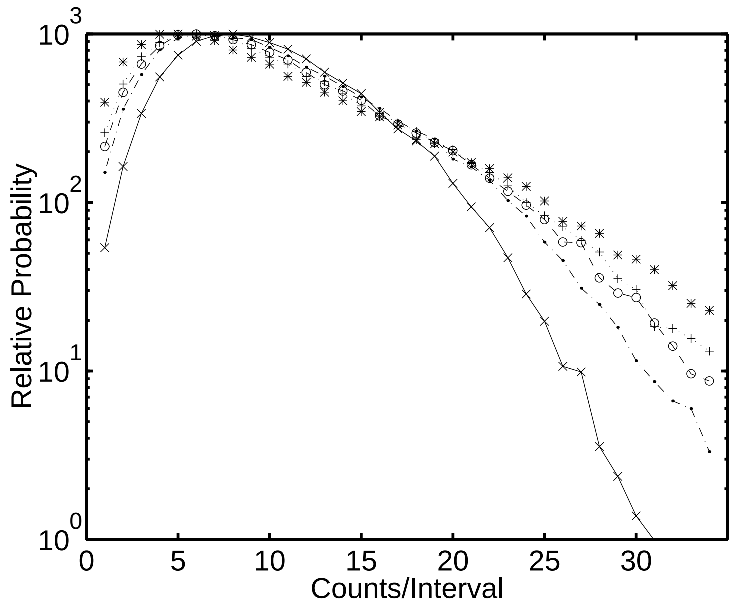

Figure 6. Poisson comparison plot simulations for $\alpha = 2$ and $\beta = 20$ for different avalanche speed parameter (see text) γ: $\gamma = 4$ ($\times$), $\gamma = 8$ ($\cdot$), $\gamma = 12$ ($\circ$), $\gamma = 16$ (+), $\gamma = \infty$ ($\divideontimes$).

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